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Solution of Aircraft Inverse Problems by Local Optimization

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A feedforward control technique is presented for the determination of the input/output map associated to the inverse problem of the simulated motion of an aircraft. The procedure is particularly suitable in the redundant cases, which are reduced to nominal ones by imposing physically reasonable constraints on the state variables through a local optimization algorithm. Comparisons with already available solutions are shown together with some significant applications.

Nomenclature

$\boldsymbol{A}$	= Jacobian matrix
$\boldsymbol{H}$	= Hessian matrix
$\boldsymbol{I}$	= unit matrix
r	= yaw velocity
s	= cost function
t	= time
$\boldsymbol{u}$	= control vector
V	= velocity modulus
x, y, z	= inertial coordinates
$\boldsymbol{x}$	= state vector
$\boldsymbol{y}$	= output vector
α	= angle of attack
β	= sideslip angle
δ_a	= aileron angle
δ_e	= elevator angle
δ_r	= rudder angle
δ_{th}	= thrust level
Δt_c	= time of constant control
Δt_i	= integration step
Δ_{t_o}	= output delay
η	= step length parameter
ξ	= direction of search
ϕ, θ, ψ	= Euler angles
$\dagger$	= generalized inverse matrix

Introduction

THE flight control of an airplane in a general motion was interpreted as an inverse dynamic problem in relatively recent years. We would like to remember here the pioneer works by Meyer and Cicolani,[1] Kato and Sugiura,[2] and Vukobratovič and Stojič.[3] In particular, to the authors' knowledge, in Ref. 2 this question was, for the first time, dealt with by following a differential approach that is, however, subjected to significant limitations in practical applications and is usually very cumbersome. The increasing interest in aeronautical inverse control problems was thereafter demonstrated by noticeable contributions as, for instance, those presented by Sentoh and Bryson[4] and by Hess and co-workers,[5,6] where different solution procedures were proposed for applications that stem from the analysis and design of automatic control systems. Applications concerning inverse maneuvers of helicopters have also been considered in Refs. 5 and 7. In the latter case a solution method similar to the one adopted in Ref. 2 was used. We should also mention the influence of the progress in inverse problem solving made in robotics and automation where systems with a large number of degrees of freedom are considered.[8] However, in robotics, only kinematic relations are usually required to be satisfied, instead of the nonlinear dynamic equations required in aeronautics.

Nowadays an even more promising area of research is opened by the cogent need for studies concerning aircraft maneuvers and supermaneuvers that involve great-amplitude, highly nonlinear motions and require rather complex control sequences where use is made of nonconventional aerodynamic control surfaces and of a vectored thrust as control inputs. In this respect, an accurate evaluation of the feasibility of the desired maneuvers should always be carried out before a first step toward the control problem is made. An example of such an evaluation is Ref. 9. This should be kept in mind not only from the point of view of the aircraft aerodynamic and structural characteristics but also when the ability of the controls to exert the proper actions is taken into account.[4]

A clear introduction to the inverse and optimal controls for desired outputs is in Ref. 4, where attention is particularly paid to the feedforward control. In that paper, the inverse control solution is found for the so-called nominal problem, where the number of inputs equals the number of constrained outputs. This is obtained by specifying the initial and final state and by minimizing an integral performance index that is based on the square deviation of the actual state from the desired one with constraints on the controls. In this global optimization procedure, the local values of the state variables during the time history are not exactly recovered when the equations of motion are integrated with the obtained control laws. On the other hand, this procedure prevents unfeasible control actions from being requested.

Another general technique is proposed in Refs. 5 and 6, where the inverse problem is reformulated as part of an integration process. The input-output inverse relationship is determined by an iterative procedure that consists of a forward integration algorithm. The step control inputs are changed in such a fashion as to drive to zero a function that represents the deviation of the solution from the assigned output. Applications of the method were presented for both the nominal and redundant cases. In the latter situation the number of inputs r is greater than the number of constrained outputs m, the difference $r - m$ being termed degree of redundancy. In that approach, advantage is taken of a procedure where the mapping of the output vector $\boldsymbol{y}$ from the input vector $\boldsymbol{u}$ is obtained through a numerical algorithm where the partial derivatives $\partial y_i / \partial u_k$ instead of the derivatives with respect to the time, are directly evaluated. This fact makes the problem of the inverse simulation more manageable even though the time intervenes through the dynamics of the airplane. According to the authors of those papers some kind of filtering was required for smoothing high-frequency oscillations, which they explained as the probable effects of multiple local minima of the error function.

As a further example of how a redundant problem may be managed, we recall the paper by Snell et al.,[10] where the flight control system of a supermaneuverable aircraft is designed by a nonlinear inversion technique. Once the desired dynamics is assigned to the system state, the application of the inversion method requires the number of states and control inputs to be the same. This condition

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is realized, in Ref. 10, by decoupling fast and slow control actions in a two-time-scale analysis of the aircraft dynamics.

The method that is now presented follows, to some extent, the method of Ref. 5, since the numerical procedure is still based on the direct calculation of the derivatives $\partial y_i/\partial u_k$ and its main intent is simulation. In particular, the nominal cases are dealt with in the same way as done there with some minor differences concerning the implemented algorithms. Frequent difficulties are reported in the literature to arise in the redundant cases and involve the opportunity of a digital input filter[5] and in some cases prevent a converged solution to be obtained. Here the solution is sought for by means of a local optimization as a way to avoid those difficulties. This formulation is relatively simple, although global optimality[4] cannot be guaranteed. A further drawback of local optimization is that the tracking algorithms generate nonconservative motions[11] in the sense that sometimes a closed path in output space does not yield a closed path in control space. This problem does not appear in global optimization, which however requires heavy computations and does not permit real-time implementation at the present time.

When the redundant cases corresponding to the most usual maneuvers are considered, instead of introducing a generalized inverse matrix, as in Refs. 5 and 6, here a suitable object function is defined that optimizes some performance, according to a proposal for redundant robots.[11,12] In this paper two ways are followed in order to carry out the optimization process. The first one involves the application of the classical finite difference Newton method. The second procedure is based on a sequential quadratic programming (SQP) algorithm and proved to be very efficient in the majority of the application-oriented calculations.

Analysis

We consider the system

$$\dot{x} = f(x, u), \qquad x(0) = x_0 \tag{1}$$

$$y = g(x, u) \tag{2}$$

where the state vector $x \in R^n/u \in R^r$ and $y \in R^m$. Let u be a step-constant function, i.e., $u(t) = u_j$, $t_{j-1} < t \le t_j$, so that, for a known x_{j-1}, numerical integration of Eq. (1) gives

$$x_j = F(x_{j-1}, u_j) \tag{3}$$

From Eqs. (2) and (3) we have

$$y_j = g(x_j, u_j) = h(x_{j-1}, u_j) \tag{4}$$

The inverse problem consists of finding the discretized input u_j^* at each time step for an assigned output y_j^D. Sufficient conditions for the implicit function

$$y_j^D = h(x_{j-1}, u_j^*) \tag{5}$$

to be inverted are[3]

$$r \ge m, \qquad \mathrm{rank}\left(\frac{\partial h}{\partial u_j}\right) = m \tag{6}$$

Under conditions (6), Eq. (5) can be expanded as

$$y_j^D = h\left(x_{j-1}, u_j^{(0)}\right) + A_0\left(u_j^* - u_j^{(0)}\right) + o\left(\left\|u_j^* - u_j^{(0)}\right\|\right)^2 \tag{7}$$

where $u_j^{(0)}$ is a first-guess value for u_j^* and the Jacobian

$$A_0 = A\left(x_{j-1}, u_j^{(0)}\right) = \left.\frac{\partial h}{\partial u_j}\right|_{x_{j-1}, u_j^{(0)}}$$

is numerically evaluated by a finite difference technique. Neglecting the second and higher order terms in Eq. (7), we have

$$u_j^* = u_j^{(0)} + A_0^\dagger\left[y_j^D - h\left(x_{j-1}, u_j^{(0)}\right)\right] + v \tag{8}$$

where $A_0^\dagger$ is the generalized inverse matrix of A_0 and v is a vector belonging to the nucleus of A_0, i.e., $v = (I - A_0^\dagger A_0)S$, where I is the unit matrix and $S \in R^r$ is an arbitrary vector projected onto the null space of A_0. Since, from Eq. (6), $r \ge m$ and A_0 has maximum rank, one of the solutions of the system $A_0\Xi = b$, $\Xi = u_j^* - u_j^{(0)}$, $b = y_j^D - h(x_{j-1}, u_j^{(0)})$ is, of course, $\Xi = A_0^\dagger b$ and the remaining ones may be obtained by assigning different values to S.

An iterative procedure can then be formulated as

$$u_j^{k+1} = u_j^{(k)} + A_k^\dagger\left[y_j^D - h\left(x_{j-1}, u_j^{(k)}\right)\right] + \left(I - A_k^\dagger A_k\right)S \tag{9}$$

Depending on S we can select different logics for the solution of the inverse problem. In particular, in Ref. 5 the redundant cases are solved by taking $S = 0$ and by computing $A_k^\dagger$ as a Moore-Penrose generalized inverse matrix (pseudoinverse matrix). When applied to Eq. (9), the pseudoinverse matrix gives a solution of minimum Euclidean norm $\|u_j^{(k+1)} - u_j^{(k)}\|$. As a consequence, the values of the control inputs at convergence will strongly depend on the first guess $u_j^{(0)}$, as will be shown later.

In the present study we associate to Eq. (5) a condition expressed as the minimum of an assigned scalar, positive-semidefinite function $s(x_{j-1}, u_j)$. This is equivalent to assigning a value $S \neq 0$.

According to what was already observed in Ref. 11, a cost function that depends on some states of the system enables one to optimize the desired constraints without introducing these states in the assigned output vector and without reducing the degree of redundancy of the problem. In this way a well-tailored solution can be obtained by a suitable cost function. In fact, a reduction of the degrees of freedom requires a more stringent evaluation of the achievability of the states that intervene in the desired output.

1) The fundamental set of equations can be expressed as

$$\begin{cases} c + A^T\lambda^* = 0 \\ h(x_{j-1}, u_j^*) - y_j^D = 0 \end{cases} \tag{10}$$

where λ indicates Lagrange multipliers and $c = (\partial s/\partial u_j)_{x_{j-1}, u_j}$. This system represents a set of $m + r$ equations for $m + r$ unknowns (r control inputs and m Lagrange multipliers). By application of the Newton's method [as for Eq. (7)] and neglecting higher order terms, we have, in matrix form,

$$\left(u_j^{k+1}, \lambda^{(k+1)}\right)^T = \left(u_j^{(k)}, \lambda^{(k)}\right)^T - B^{-1}\begin{bmatrix} c\left(x_{j-1}, u_j^{(k)}\right) + A_k^T\lambda^{(k)} \\ h\left(x_{j-1}, u_j^{(k)}\right) - y_j^D \end{bmatrix} \tag{11}$$

The symmetric matrix B is given as

$$B = \begin{bmatrix} H_k + \left.\dfrac{\partial A_k^T\lambda^{(k)}}{\partial u_j}\right|_{x_{j-1}, u_j^{(k)}} & A_k^T \\ A_k & 0 \end{bmatrix}$$

and the Hessian matrix $H_k = \partial c/\partial u_j|_{x_{j-1}, u_j^{(k)}}$. As for the inversion of B, it can be shown that its determinant is nonzero if the rank of A is maximum and the first element of the first row has nonzero determinant.

2) The SQP consists of solving a quadratic programming (QP) subproblem and of updating a Hessian matrix. When the constrained optimization problem represented by the minimization of the performance criterion $s(x_{j-1}, u_j)$ subjected to Eq. (5) is pursued by the SQP method,[13–15] a quadratic approximation of the Lagrangian function $L(u, \lambda)$ is considered. The QP subproblem is to find the minimum of $\frac{1}{2}\xi^T H_k \xi + c^T\xi$ that represents the second-order approximation of the cost function s. Here the matrix H_k is a positive-definite approximation of H, the Hessian matrix of the Lagrangian function. Also, ξ is the direction of search. The solution leads to

$$u_j^{(k+1)} = u_j^{(k)} + \eta_k \xi_k$$

where η is then determined by a suitable line search procedure. Here, H_k is recursively calculated using a modified Broyden, Fletcher, Goldfarb and Shanno (BFGS) quasi-Newton update to incorporate curvature information obtained in the move from $u_j^{(k)}$ to $u_j^{(k+1)}$. The BFGS algorithm is fully described in Ref. 16.

Remark 1. A major point to be considered concerns the behavior of the solution right after the initial assumed equilibrium state is perturbed by the control actions. Depending on the time interval Δt_c during which the control actions are assumed to be constant and on the integration step of the equations of motion, the insurgence of the modes that are typical of the considered airplane can affect the convergence and accuracy of the results. This can probably explain the filtering adopted by some authors. Recall, in particular, that the aircraft can be modeled as a two-time-scale system when perturbed from equilibrium in the longitudinal plane.[17] Thus, for the applications, instead of imposing the value of $y^D(t_{j-1} + \Delta t_c)$, we imposed the value $y^D(t_{j-1} + \Delta t_o)$, with $\Delta t_o \geq \Delta t_c$, where Δt_o is an appropriate delay such that the short period, which is excited by each step action of the elevator, subsides. This means that one will not obtain the desired y^D at the time $t_j = t_{j-1} + \Delta t_c$ but at the time $t_{j-1} + \Delta t_o$ unless, of course, an output constant in time is required. The difference $\Delta t_o - \Delta t_c$ will be smaller the shorter is the subsidence. This can be done explicitly at least in the longitudinal dynamics. In the more general case, care has to be paid in empirically selecting appropriate values of Δt_o and Δt_c, as suggested in a recent note and in its comment.[18,19]

Remark 2. In the redundant cases a cost function

$$s = \tfrac{1}{2}\boldsymbol{u}_j^T \boldsymbol{V} \boldsymbol{u}_j + \boldsymbol{p}^T \boldsymbol{u}_j \tag{12}$$

can be defined, where $\boldsymbol{V} \in R^{r\times r}$ is a positive-semidefinite matrix and $\boldsymbol{p} \in R^r$ is a vector, both quantities depending, in case, on $\boldsymbol{x}_{j-1}$. Equation (7), neglecting terms of order 2 and higher, is written at each iteration as

$$\boldsymbol{y}_j^D - \boldsymbol{h}\left(\boldsymbol{x}_{j-1}, \boldsymbol{u}_j^{(k)}\right) = \boldsymbol{A}_k\left(\boldsymbol{u}_j^{(k+1)} - \boldsymbol{u}_j^{(k)}\right) \tag{13}$$

The set of equations (13) together with the local objective function (12) is thus formulated as an optimization problem, the solution to which is[11]

$$\boldsymbol{u}_j^{(k+1)} = \boldsymbol{u}_j^{(k)} + \boldsymbol{A}_v^\dagger\left[\boldsymbol{y}_j^D - \boldsymbol{h}\left(\boldsymbol{x}_{j-1}, \boldsymbol{u}_j^{(k)}\right)\right] + (\boldsymbol{I} - \boldsymbol{A}_v^\dagger \boldsymbol{A})(-\boldsymbol{V}^{-1}\boldsymbol{p})$$

where $\boldsymbol{A}_v^\dagger = \boldsymbol{V}^{-1}\boldsymbol{A}_k^T(\boldsymbol{A}_k\boldsymbol{V}^{-1}\boldsymbol{A}_k^T)^{-1}$ is called a weighted pseudoinverse matrix. This solution is analytical and explicit, and the procedure can deal effectively with control limits. With respect to this method we introduce a more general cost function wherein constraints on the state variables of the system may appear.

Results

A number of nominal and redundant problems were solved that relate to different aircraft maneuvers, some of them in order to compare our results with those obtained by different authors. In any case, since what we present here is aimed at solving redundant problems, we note that in the existing literature only few cases concerning these situations were dealt with. For all the tests the initial equilibrium conditions were evaluated through the solution of an optimization problem by introducing a cost function with penalties assigned to the time derivatives of the state parameters, as suggested in Ref. 20. The implementation of the SQP algorithm was carried out by using the module E04UCF of the NAG Fortran Library.[21]

An opportunity offered by this last technique is the enforcement of inequality constraints. This provides an effective means for assuring the feasibility of the control actions.

Case 1 (redundant). The aircraft F-4C is the same as was considered in Ref. 5 and the initial flight condition is level flight at an altitude of 4572 m and Mach number 0.9. This corresponds to an unstable equilibrium situation due to the positiveness of one of the two real roots of the phugoid mode. The control inputs are δ_e, δ_a, δ_r, and δ_{th}. We consider an aileron roll with angular velocity along the x-body axis, with constraints on the following three outputs:

$$\left.\begin{aligned} V(t) &= 290 \text{ m/s} \\ \beta(t) &= 0 \end{aligned}\right\} \qquad \forall t \geq 0$$

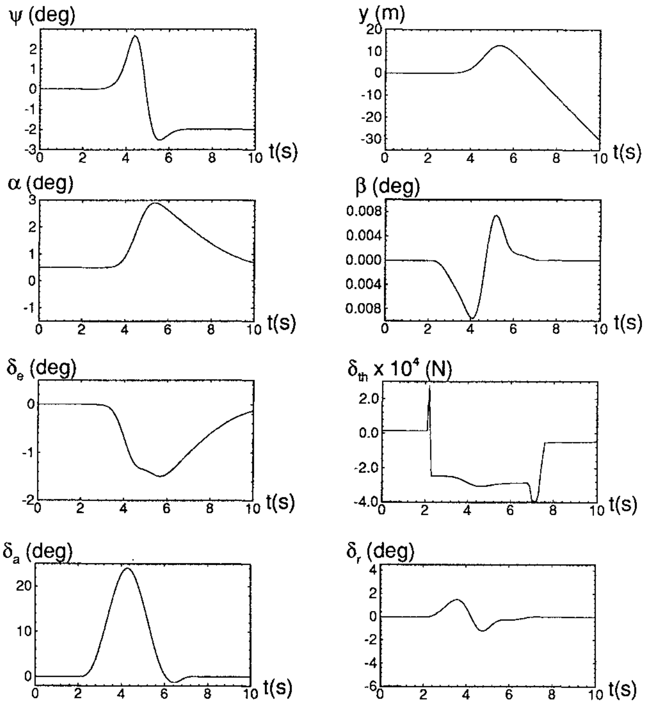

Fig. 1 Case 1: F-4C output and control variables. Newton's method with $S = 0$.

$$\phi(t) = 0, \qquad 0 \leq t \leq 2 \text{ s}$$

$$\phi(t) = 2\pi\{\cos[3\pi(t-2)/5] - 9\cos[\pi(t-2)/5] + 8\}/16 \text{ rad}, \quad 2 \leq t \leq 7 \text{ s}$$

$$\phi(t) = 2\pi, \qquad t \geq 7 \text{ s}$$

The degree of redundancy is 1 and the computations were performed by taking a value Δt_o equal to 0.6 s, whereas the integration step Δt_i of the equations of motion was equal to 0.06. Furthermore, the time of constant control Δt_c was taken equal to 0.06 s. When we applied the first procedure for $S = 0$ we obtained the solutions shown in Fig. 1, where y is the lateral displacement. They are very close to Hess et al.'s results (not shown) except for the δ_{th} response. However, the thrust variations are very small in both cases.

When we introduce time-dependent coefficients in a cost function expressed by

$$s = [1 - \gamma(t)]\alpha^2 + \gamma(t)\theta^2$$

with

$$\gamma(t) = 0, \qquad 0 < t \leq 2$$

$$\gamma(t) = \{\cos[3\pi(t-2)/8] - 9\cos[\pi(t-2)/8] + 8\}/16, \quad 2 \leq t \leq 10 \text{ s}$$

and run the same procedure but now for $S \neq 0$, we have the results of Fig. 2. The peculiar cost function introduces time-dependent penalty coefficients in order to effectively drive the solution toward the reattainment of equilibrium by including two state variables. This fact is equivalent to constrain α and θ by a penalty index without changing the degree of redundancy. In particular, the thrust does change with respect to the preceding case, but the lateral motion almost vanishes. The final change of altitude, not represented, is still of 46 m, as it is for the case of Fig. 1 and for Hess et al.'s data.[5]

We note that in the case of Fig. 1 our solution (although slightly) differs from that of Ref. 5 as a consequence of a number of possible reasons as, for instance, numerical errors and different choices of the characteristic time intervals. Moreover from Fig. 2 we observe that introducing that cost function, where a performance function $\gamma(t)$ appears as a constraint on a combination of state variables, can strongly help in obtaining an assigned final state and in so doing

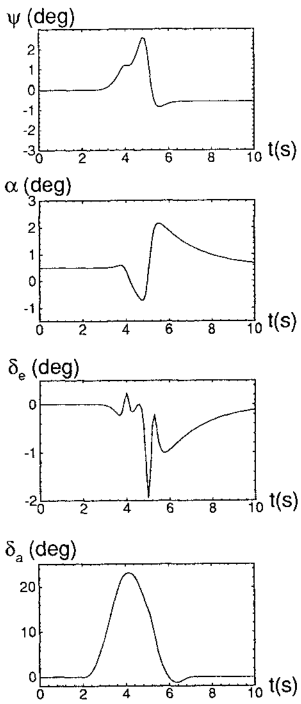

Fig. 2 Case 1: F-4C output and control variables. Newton's method with $S \neq 0$.

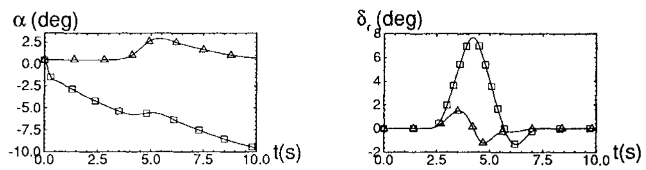

Fig. 3 Case 1: Effect of initial guess $u_j^{(0)}$: □, $u_j^{(0)} = 0.1$; △, $u_j^{(0)} = u_{j-1}^*$.

meeting some of the observations of Ref. 4. Note that the degree of redundancy is reduced only for $\gamma = 0, 1$.

As we already said, when use of the generalized inverse is made, the influence of assuming different values of $u_j^{(0)}$ can be sizable. In this respect, when case 1 is considered with $S = 0$, as in Ref. 5, Fig. 3 shows some results for $u_j^{(0)} = 0.1$ and for $u_j^{(0)} = u_{j-1}^*$. For the sake of conciseness, only α and δ_r vs. t are reported.

Case 2 (redundant). The maneuver is an aileron roll as in case 1, with four control variables but with only two assigned outputs, namely speed and roll angle, which are constrained as before. The degree of redundancy is consequently 2. When the F-4C was considered, for $S = 0$ we were not able to recover the results of Ref. 5 and our solution is not very dissimilar with respect to the solution of case 1 and will not be reported here. When a cost function was assumed,

$$s = 10^{-18}(\dot{\delta}_{\text{th}})^8 + 10^3 r^2 + 10^{-4}\delta_r^2$$

the results shown in Fig. 4 were obtained by the SQP optimization with $\Delta t_c = 0.1$ s, $\Delta t_i = 0.01$, and $\Delta t_o = 0.6$ s. The numerical coefficient of the thrust rate is understandable when we consider the very steep variation of thrust itself and was empirically determined, as were the other coefficients, with the aim of preventing unrealistic values of α.

In comparison with the data in Ref. 5 the more significant differences are connected with the final values of variables such as the angle of attack and the Euler angle ψ, reported in Fig. 4. In the present solution the variations of α are always positive and small and tend to vanish whereas ψ initially increases and then decreases below the initial value. The pitch angle is well controlled.

Case 3 (nominal). This case was run in order to show the effectiveness of the line search method of the SQP code. For a model of the aircraft F-16[20] we considered an intentional change of the sideslip angle whereas the other state variables are kept as constant as possible. We were not able to find a converged solution to this

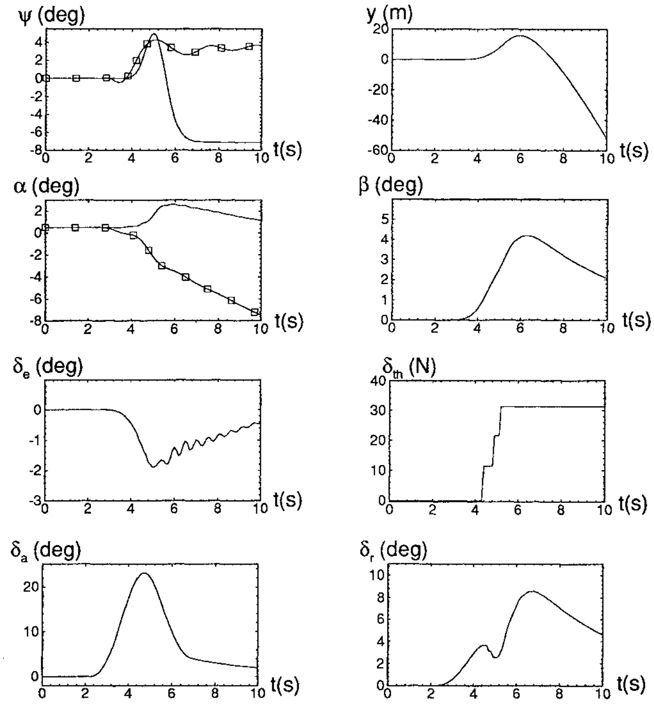

Fig. 4 Case 2: F-4C output and control variables: SQP method with $S \neq 0$; □, Ref. 5.

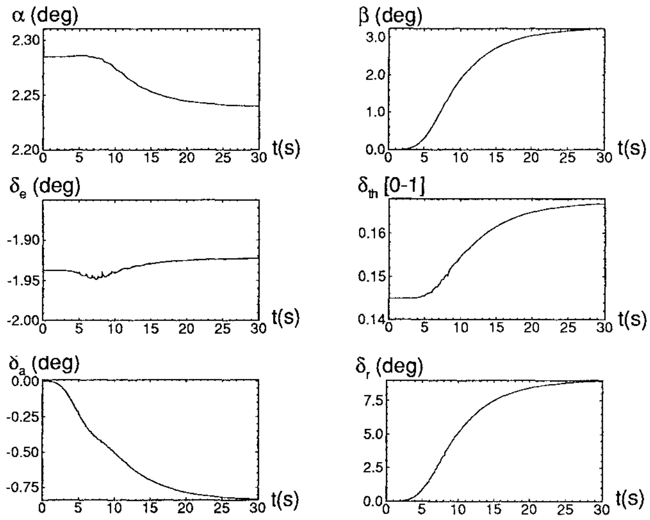

Fig. 5 Case 3: F-16 output and control variables.

problem by means of our first optimization procedure. The solution was found instead by the second optimization procedure for $s = \text{const}$, $\Delta t_c = 0.1$ s, $\Delta t_i = 0.1$, and $\Delta t_o = 1.5$ s. The number and type of controls is the same as in Case 2 but the maneuver is performed by changing the roll angle according to the law

$$\phi(t) = \pi[\cos(3\pi t/5) - 9\cos(\pi t/5) + 8]/288 \text{ rad}, \quad 0 \leq t \leq 5 \text{ s}$$

$$\phi(t) = \pi/18, \qquad t \geq 5 \text{ s}$$

Furthermore we impose

$$\left.\begin{aligned} V(t) &= 152 \text{ m/s} \\ \psi(t) &= 0 \\ z(t) &= 0 \end{aligned}\right\} \qquad \forall t \geq 0$$

Figure 5 shows the main state variables and the controls. The convergence of the solution to the final state proved to be very good, as usually happens when the SQP method is applied to nonlinear constrained functions in comparison with penalty methods.[16]

Conclusion

The present approach to the solution of inverse redundant problems in flight mechanics showed neither necessity for filtering nor excessive sensitivity to the initial guess in all the tested cases when compared with other proposed procedures. In the redundant cases, in particular, the introduction of a minimum of a physically meaningful performance index s allows for a much better control on the convergence of the solution. Furthermore introducing cost functions wherein time-dependent penalty coefficients appear can drive the solution toward a terminal equilibrium state as obtained, for example, by means of the global optimization procedures. The difficulties met in the applications can be associated with the choice of the cost function, the laws assumed for the output variables, and the evaluation of the time-dependent sensitivity matrix of the outputs with respect to the controls.

Many other authors working on nonlinear inversion techniques applied to aircraft dynamics do not use iterative methods and may face excessive demands for control action. The method of the present paper relates the control effort to the errors between desired and active outputs, and those demands have physical meaning so that the saturation of controls can actually be avoided or postponed.

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